Axial Dispersion in Laminar Flow of Polymer Solutions through Coiled Tubes

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Synopsis

The results of an experimental study on axial dispersion in laminar flow of non-Newtonian fluids through helical coils are reported. The ranges of variables covered are $10.5 \leqslant \lambda \leqslant 220$, $0.6 \leqslant n \leqslant 1.0$, $0.1 < N_{\text{Regn}} < 140$, and $0.04 < \tau < 2.2$. The condition for the applicability of Taylor's dispersion model is also reported. It is found that coiling results in a dispersion reduced over that in a straight tube.

INTRODUCTION

Complex flow behavior of fluids flowing through coiled tubes has been drawing the attention of engineers for the last few decades. The reason for this is the mixing in the cross-sectional plane, causing an increase in the heat transfer coefficient, hence making it better heat transfer equipment, and causing a reduction in axial dispersion, which improves the performance of a flow reactor. Some papers have appeared recently[1–5] which discuss the pressure drop studies with non-Newtonian fluids in helical coils. Mashelkar and Devrajan[1,2] have solved the equations of motion for laminar non-Newtonian flow in coiled tubes using a boundary layer approximation. For a given fluid they have presented a friction factor vs. Dean number relationship and provided an experimental support to their analysis. Experimental results on the study of pressure drop in laminar flow of non-Newtonian fluids through helical coils have also been reported in Refs. 3 and 4. Recently, Mujawar and Raja Rao[5] have reported an experimental study on the pressure drop in helical coils and have suggested a new dimensionless number M, which better characterizes the helical flow. They have correlated friction factor with this new dimensionless number.

Literature on axial dispersion in helical coils have been limited to Newtonian fluids only. Erdogan and Chatwin[6] and Nunge et al.[7] derived the expressions for axial dispersion coefficient (K_c) for Newtonian fluids incorporating the molecular diffusional effect with Dean's[8] and Topokoglu's[9] velocity profiles, respectively. Both the expressions relate the dispersion coefficient with N_{Re}, N_{Sc}, N_{Bo}, and λ with a difference that the expression reported by Nunge et al. includes additional terms involving N_{Re}^2 and N_{Re}^0 against only N_{Re}^4 in Erdogan and Chatwin's[6] expression, which describes the effect of coiling on dispersion. Nigam and Vasudeva[10] have experimentally shown that the analysis of Erdogan and Chatwin is quite accurate in predicting the minimum Dean number at which the effect of curvature on axial dispersion is significant, but is inadequate in predicting the extent of reduction. Trivedi and Vasudeva[11] in their experimental

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study with Newtonian fluids observed that under the conditions of applicability of dispersion model the reduction in axial dispersion due to coiling can be of the order of 2–500-fold, depending upon the system parameters. They have given the criterion for the applicability of Taylor's[12] dispersion model in case of Newtonian flow through coils as

$$\tau > 6\, N_{\mathrm{Re}}^{-1} \tag{1}$$

Sakra et al.,[13] based on their experimental study on laminar dispersion for the helical flow through rectangular channels, reported a considerable reduction in dispersion number with an increase in the Reynolds number.

The knowledge of dispersion phenomena in helical flow of non-Newtonian fluids is of importance in the biomedical field, design of flow reactors for biological systems, and many other industrial applications. Despite its importance in different fields, practically no information is available on the dispersion of polymer solutions in coiled tubes except the work of Singh and Nigam,[14] which is also based on very limited experimental observations. In the present study more than 100 experiments have been carried out to cover a wide range of system parameter values of practical interest. Based on the present study a new criterion for the applicability of axial dispersion model has been suggested.

THEORY

Taylor's[12] one-dimensional axial dispersion model can be written as

$$\frac{\partial C}{\partial \theta} = \frac{D}{\bar{u}L}\frac{\partial^2 C}{\partial X^2} - \frac{\partial C}{\partial X} \tag{2}$$

where C = cross-sectional mean concentration at the outlet, $\bar{u}$ = average linear velocity, L = reactor length, X = dimensionless axial distance, D = effective diffusion coefficient, and θ = dimensionless time. The solution of eq. (2) for different boundary conditions has been compiled in Ref. 15.

In the present study the solution for doubly infinite boundary condition has been used because it is simple to use and does not introduce any error for large values of Peclet number encountered in the present study. The solution of Eq. (2) can be written as[16]

$$\mathrm{F} = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{1-\theta}{2\sqrt{\theta(D/\bar{u}L)}}\right)\right] \tag{3}$$

Ananthkrishna et al.[17] stressed the need of distinction between cross-sectional mean concentration and bulk mean concentration, while Ferrel and Himmelblau[18] showed that for $\tau > 2.0$ these two are essentially the same. Recently Nigam and Vasudeva[19] have shown that for the reactor performance the concentration term in dispersion model may be assumed as bulk mean concentration.

Fan and Hwang[20] showed that the one-dimensional Taylor's model is also valid for the flow of power law fluids in straight tubes if the condition

$$\tau \gg 0.0682\,(n' + 3)/(n' + 1) \tag{4}$$

is satisfied, and the expression for dispersion number can be given as

$$\frac{D}{\bar{u}L} = \frac{1}{2\,(n' + 3)(n' + 5)\tau} \tag{5}$$

where n' = inverse of flow behavior index and τ = dimensionless characteristic time ($\bar{t}D_m/a^2$). The correctness of the Fan and Hwang analysis has been experimentally verified by a number of investigators.[21–23] The applicability of Taylor's dispersion model for the flow of non-Newtonian fluids in straight tubes implies that the study of axial dispersion in laminar flow of non-Newtonian fluids through helical coils is comprised of investigating the condition where the Taylor's dispersion model is applicable and of relating the effective diffusion coefficient with system parameters.

EXPERIMENTAL

Six coils of different curvature ratio (λ) were used in the present study. The coils were prepared by coiling thick-walled flexible PVC tubing over cylindrical bases. The dimensions of the coils are reported in Table I. Aqueous solutions of sodium carboxy methyl cellulose (CMC) of different concentrations (in the range 0.0–2.0 g/L) were used as flowing media. A capillary tube viscometer[24] was used to measure rheological properties of CMC solutions. Molecular diffusion coefficients of tracer (congo red) in CMC solutions were experimentally determined by measuring the extent of dispersion in straight tubes.[21–23] Details of the method are reported elsewhere.[22] Experimentally determined values of rheological properties of CMC solutions and molecular diffusion coefficient of the tracer are given in Table II.

Experimental setup consists of two constant head reservoirs containing solvent and tracer solutions connected to the helical coil under study through a threeway stop cock. Experimental procedure involved changing the flowing fluid from

TABLE I
Dimensions of Helical Coils

Coil no.	Tube diam d_t, cm	Tube wall thickness, cm	Tube length L, cm	Coil diam d_c, cm	Coil pitch h, cm	λ
1	0.301	0.15	2340.0	3.16	0.601	10.5
2	0.301	0.15	1408.1	3.16	0.601	10.5
3	0.31	0.15	2443.0	9.29	0.61	30.0
4	0.31	0.15	2443.0	31.31	0.61	101.0
5	0.31	0.15	4234.4	31.18	0.61	101.0
6	0.31	0.15	2443.0	68.2	0.61	220.0

TABLE II
System Characteristics

Solvent	Density, g/L	Flow behavior index, n	Consistency index K_s, g·s^{n-2}/cm	Molecular diffusivity x 10^6, cm^2/s
Water	1.0	1.0	0.01	3.70
0.1 g/L CMC in water	1.0	0.915	0.022	3.81
0.5 g/L CMC in water	1.01	0.725	0.244	5.35
1.0 g/L CMC in water	1.01	0.647	0.736	5.63
2.0 g/L CMC in water	1.02	0.607	1.644	5.99

solvent to tracer solution at a predetermined instant and flow rate. About 20 samples at suitable intervals were collected at the outlet over the duration where the concentration of the outgoing stream changes from minimum to maximum. The sampling time was always kept less than 1/50 of the residence time $\bar{t}$. The optical density of the samples was measured using an ELICO colorimeter at a wave length of 520 μm. The linear relationship between optical density and the dye concentration over the employed concentration range facilitated the direct use of optical density to obtain the F curve. This technique for determining response to step input (F-curve) is well established and was employed in our previous publications.[10,14,15,22,25]

RESULTS AND DISCUSSION

Step response experiments were carried out in all the six coils with aqueous CMC solutions of the five different concentrations (corresponding values of power law index being 0.6–1.0). The range of generalized Reynolds numbers covered was from 0.1 to 140, which corresponds to the Dean number range 0.006–45. More than 100 experiments were performed, out of which, for 65 response curves, axial dispersion model was found to be valid.

The criteria for the validity of Taylor's dispersion model for the experimentally determined response curves have been discussed in detail by Trivedi and Vasudeva.[11] In the present study the value of $D/\bar{u}L$ for an experimentally obtained F curve was computed by matching it with the theoretical F curves defined by eq. (3) for different assumed values of $D/\bar{u}L$. The criterion involved for the best fit was minimum $\Sigma|F_{\text{exp}} - F_{\text{theo}}|$. If the value of $\Sigma_{\min}|F_{\text{exp}} - F_{\text{theo}}|$ calculated at the intervals of 0.05θ was more than 0.5, which implies that more than 1.25% fluid is assigned incorrect residence time by the model, it was rejected.

Effect of Curvature Ratio and Power Law Index on Response Curves

Figure 1 shows the effect of curvature ratio (λ) on response curves, obtained for a fixed value of the flow behavior index ($n = 0.64$) and constant value of τ (=0.45). From Figure 1 it can be seen that the response curve becomes narrower as λ decreases. This is expected due to the increase in secondary flow with reduction in λ. A similar trend was observed for other values of the flow behavior index, but it was noticed that this phenomenon is more significant at lower values of the power law index.

The effect of the flow behaviour index on response curves for fixed values of curvature ratio ($\lambda = 10.5$) and τ (=0.26) is shown in Figure 2, which reveals the narrowing of residence time distribution (hence reduction in $D/\bar{u}L$) with a decrease in the power law index (n). This trend is obvious because of the flattening of the axial velocity profile with a decrease in the flow behaviour index.[24]

Axial Dispersion

The $D/\bar{u}L$ values were computed for the cases where the dispersion model was valid. No trend of $D/\bar{u}L$ values with N_{Regn} for any of the six coils was observed for a given value of the flow behavior index and curvature ratio. Similar scattering with effective Bodenstein number ($\bar{u}d_t/D$) have been reported for New-

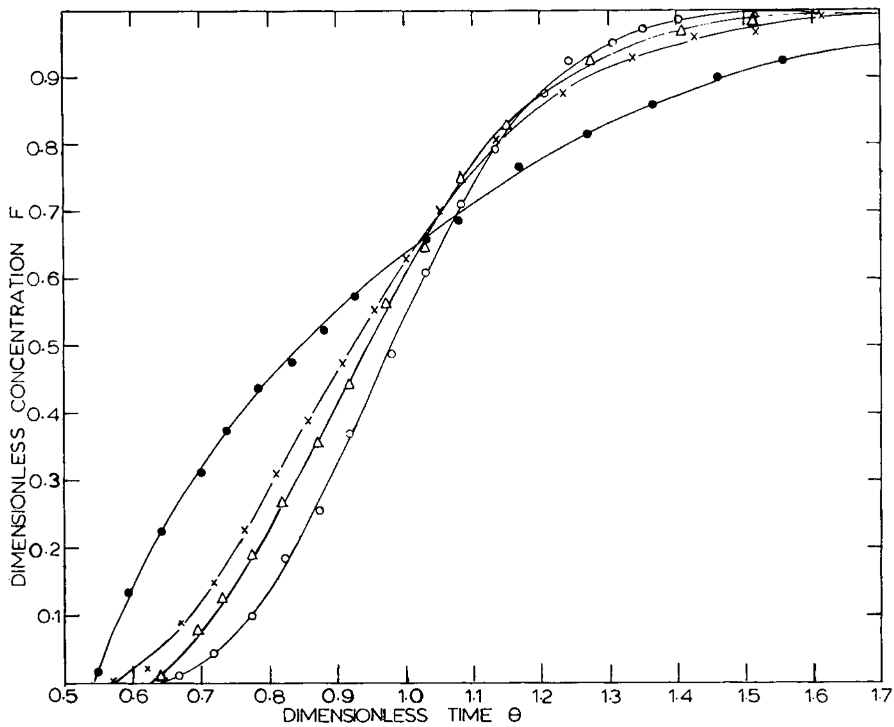

Fig. 1. Effect of curvature ratio on response curves: n = 0.64, τ = 0.45; (O) λ = 10.5; (△) λ = 30.0; (X) λ = 101.0; (●) λ = 220.0.

tonian fluids in coils.[11,26] Since the dispersion number and the effective Bodenstein number are related through L/d_t, which does not involve any of the operating parameters, the scatter obtained with $D/\bar{u}L$ in the present study is comparable to those reported with the effective Bodenstein number.

Erdogan and Chatwin[6] incorporating diffusional effects with Dean's[8] velocity profile have shown that the extent of dispersion in coiled tubes for Newtonian fluids can be given as

$$K_C = \frac{DD_m}{\bar{u}^2 d_t^2} = \left(\frac{1}{N_{\mathrm{Bo}}^2} + \frac{1}{192}\right) + \frac{4N_{\mathrm{Re}}^4 \lambda^{-2}}{576 \times 160}\left(\frac{2569}{15{,}840} N_{\mathrm{Sc}}^2 + \frac{109}{43{,}200}\right) \tag{6}$$

In the present study K_C was computed using the best fit value of $D/\bar{u}L$, and the results are plotted against N_{Regn} in Figure 3, which shows the effect of curvature ratio on axial dispersion for a given value of flow behavior index. From Figure 3 it is also clear that for a fixed value of the flow behavior index the experimental results observe a similar trend (with N_{Regn} and λ), as reported by Trivedi and Vasudeva[11] for Newtonian fluids. Despite this similarity the different sets of curves for different values of the flow behavior index suggests that use of N_{Regn} is not sufficient to define axial dispersion in the helical flow of non-Newtonian fluids. In a recent publication, Muzawar and Raja Rao[5] have suggested that the flow of non-Newtonian fluids in helical coils can be better characterized by a dimensionless number M. In order to examine its validity for the case of axial dispersion, the dimensionless group M was computed using their correlations. Figure 4 shows the plot K_C vs. M. Again the different sets of curves for different

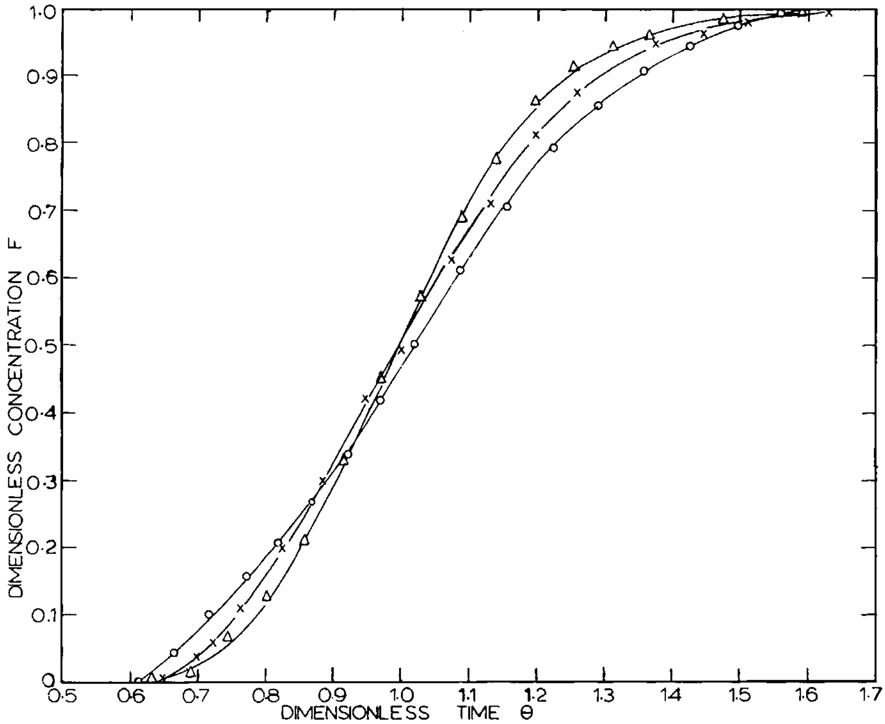

Fig. 2. Effect of flow behavior index on response curves: $\lambda = 10.5$, $\tau = 0.26$; (O) n = 1.0; (X) n = 0.72; (△) n = 0.64.

values of the flow behavior index, even for the experiments which falls within the ranges $14 < \lambda < 100$, $1 < n < 0.7$, and $0.008 < K_S < 2$, for which Muzawar and Raja Rao have reported their correlations, suggests that this criterion is not of much use in characterizing the axial dispersion in coils. It also indicates that the dimensionless number M may not be a true representation of the hydrodynamics in the helical flow of non-Newtonial fluids.

It was thought intutively that $N_{\text{Regn}} \cdot N_{\text{Sc}}$ may characterize the dispersion in coils because it nullifies the deviations in N_{Regn} due to the large differences in apparent viscosities of different fluids. Hence the Schmidt number was evaluated as

$$N_{\text{Sc}} = \rho\mu_a/D_m \tag{7}$$

where μ_a, the apparent viscosity, was calculated using the relation

$$\mu_a = K_S(8\bar{u}/d_t)^{n-1} \tag{8}$$

Eq. (8) is essentially for straight tubes, but, as per the analysis reported by Mashelkar and Devrajan,[1] for $\lambda \gg 1$, the shear rates in radical directions are negligible as compared to axial shear rates and shear stress in radial plane and axial direction ($\tau_{r\phi}$) may be approximated by

$$\tau_{r\phi} \simeq K_S\ (du/dr)^n \tag{9}$$

Therefore, eq. (8) could be used to evaluate apparent viscosity in coiled tubes. It can be seen from Figure 5 that for a coil of given curvature ratio all the experimental points for different fluids can be approximated by a single curve if K_C is plotted against $N_{Regn} \cdot N_{Sc}$. From Figure 5 it may be concluded:

(i) Axial dispersion decreases with increase in the value of $N_{Regn} \cdot N_{Sc}$ for helical flow of non-Newtonian fluids.

(ii) For a fixed value of $N_{Regn} \cdot N'_{Sc}$ axial dispersion increases with increase in λ.

(iii) At lower values of $N_{Regn} \cdot N_{Sc}$, where the values of K_C are closer to that in straight tubes, the fact that the curvature has no effect on axial dispersion suggests that for $N_{Regn} \cdot N_{Sc} < 20{,}000$ fluid elements are blind to curvature and coil behaves like a straight tube.

(iv) In the range of variables covered in the present study up to 12-fold reduction in the value of K_C due to coiling was observed, depending upon the value of $N_{Re} \cdot N'_{Sc}$ and λ.

In order to examine quantitatively the dependence of K_C on λ, a logarithmic plot of K_C vs. λ was plotted for 10 different values of $N_{Regn} \cdot N_{Sc}$ in the range $1.5 \times 10^4 < N_{Regn} \cdot N_{Sc} < 3 \times 10^5$. The different slopes of all the straight lines (obtained for each value of $N_{Regn} \cdot N_{Sc}$) suggested that effect of the curvature ratio on axial dispersion is a function of $N_{Regn} \cdot N'_{Sc}$. By using Lagrangian basis functions,[27] the slope of these straight lines can be represented by the curve

$$\text{slope} = \phi(N_{Regn} \cdot N_{Sc}) = (234.29\xi - 0.075\xi^3 - 2.603\xi^2 - 345.39) \times 10^{-4} \quad (10)$$

where $\xi = (N_{Regn} \cdot N_{Sc}) \times 10^{-4}$.

Figure 6 is a plot between K_C and $N_{Regn} \cdot N_{Sc} / \lambda^{\phi(N_{Regn} \cdot N_{Sc})}$

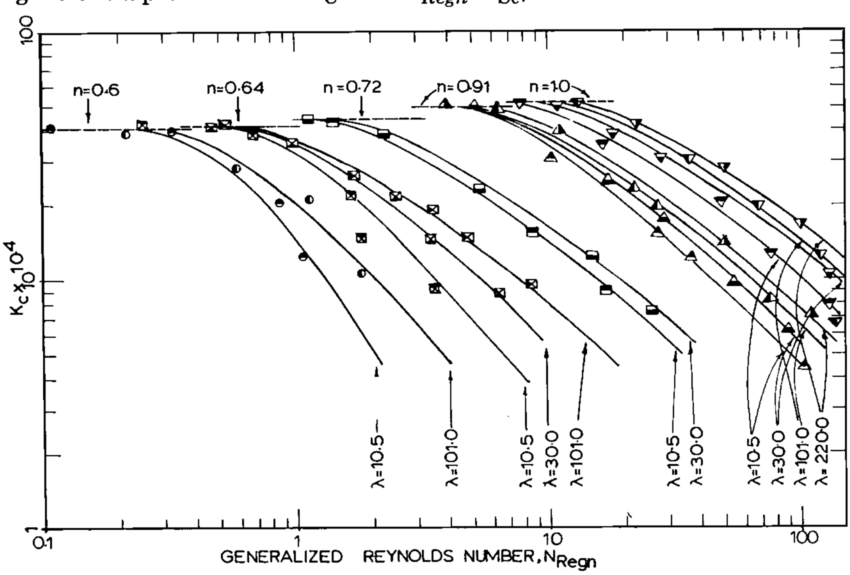

Fig. 3. K_C vs. N_{Regn}. (—) present work, (- - -) Fan and Hwang[20] (for straight tubes).

Symbols

n/λ	10.5	30.0	101	220
0.6	◒		◐	
0.64	⊠	⊠	⊠	
0.72	⊟	⊟		
0.91	▲	▲	▲	▲
1.0	▼	▼	▼	▼

As can be seen from Figure 6 that for all the six coils and for all the five fluids, the results can be represented by a single curve. The correlation can be written as

$$K_C = 1.0/8\,(n' + 3)(n' + 5) \text{ for } N_{\mathrm{Regn}}{\cdot}N_{\mathrm{Sc}} \leqslant 20{,}000 \tag{11a}$$

$$K_C \times 10^4 = 2.3344\psi^2 - 0.04416\psi^3 - 27.5418\psi + 102.8297 \tag{11b}$$

for $20{,}000 < N_{\mathrm{Regn}}{\cdot}N_{\mathrm{Sc}} < 3 \times 10^5$ where $\psi = [N_{\mathrm{Regn}}{\cdot}N_{\mathrm{Sc}}/\lambda^{\phi(N_{\mathrm{Regn}}\cdot N_{\mathrm{Sc}})}] \times 10^{-4}$

Condition for the Applicability of Dispersion Model

The essence of the theory of dispersion is to provide the condition under which axial dispersion model is valid. With this intention, when τ vs. N_{Regn} values were plotted, no distinction between experiments in which the dispersion model was applicable and the cases in which the dispersion model was not applicable could be observed. However, the criterion for the validity of dispersion model reported by Trivedi and Vasudeva[11] for the helical flow of Newtonian fluids ($\tau > 6\,N_{\mathrm{Re}}^{-1}$) was found to be perfectly correct for $n = 1$ It can be seen from Figure 1 that the curvature ratio (λ) has a strong influence on applicability of dispersion model. In view of this the values of τ and N_{De} ($=N_{\mathrm{Regn}}/\sqrt{\lambda}$) were plotted in Figure 7. It can be seen from Figure 7 that a distinction between the experiments where Taylor's dispersion model was applicable and the cases where it was not appli-

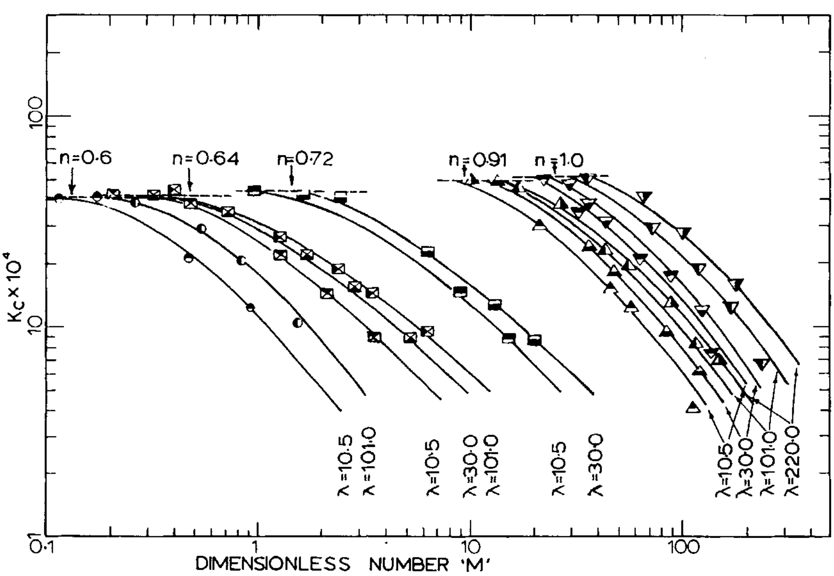

Fig. 4. K_C vs. M. For symbols, see Table IV: (—) present work, (- - -) Fan and Hwang[20] (for straight tubes).

	Symbols			
n/λ	10.5	30.0	101	220
0.6	⊖		◐	
0.64	⊠	⊠	⊠	
0.72	⊟	⊟		
0.91	▲	▲	▲	▲
1.0	▼	▼	▼	▼

cable emerged out. The condition for the validity of the dispersion model for the helical flow of non-Newtonian fluids, over the range of variables studied in the present work, can be given as

$$\tau > 0.22 N_{\mathrm{De}}^{-0.6} \tag{12}$$

CONCLUSIONS

(i) The experimentally obtained values of dispersion number have been successfully correlated in terms of $DD_m/\bar{u}^2 \cdot dt^2$ as a function of $N_{\mathrm{Regn}} \cdot N_{\mathrm{Sc}}$ and with a standard deviation of 12.6%.

(ii) The axial dispersion decreases with an increase in the value of $N_{\mathrm{Regn}} \cdot N_{\mathrm{Sc}}$ for a given coil. With a decrease in curvature ratio the axial dispersion decreases for a given value of $N_{\mathrm{Regn}} \cdot N_{\mathrm{Sc}}$.

(iii) It was observed that the new dimensionless number suggested by Muzawar and Raja Rao[5] is not of much use in characterizing the axial dispersion in laminar non-Newtonian helical flow.

(iv) An appropriate condition under which the dispersion model is likely to hold is provided.

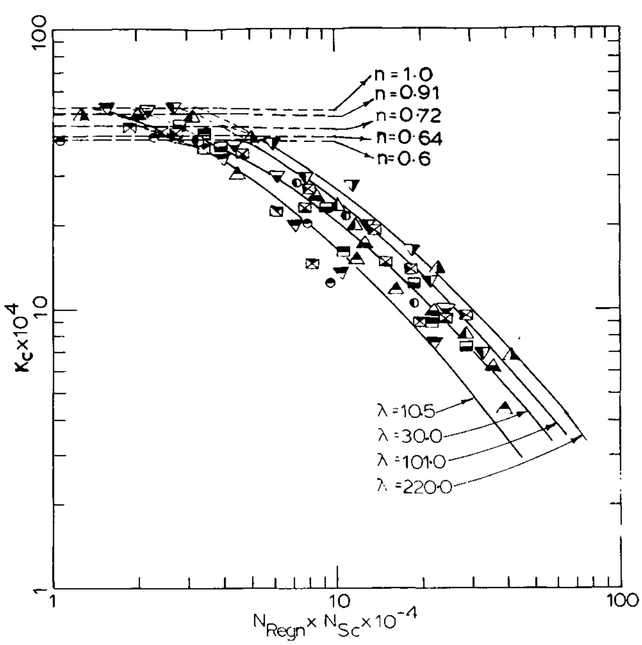

Fig. 5. K_C vs. $N_{\mathrm{Regn}} \cdot N_{\mathrm{Sc}}$. (—) present work, (- - -) Fan and Hwang[20] (for straight tubes).

Symbols

n/λ	10.5	30.0	101	220
0.6	◓		◐	
0.64	⊠	⊠	⊠	
0.72	⊟	⊟		
0.91	△	△	▲	▲
1.0	▽	▽	▼	▼

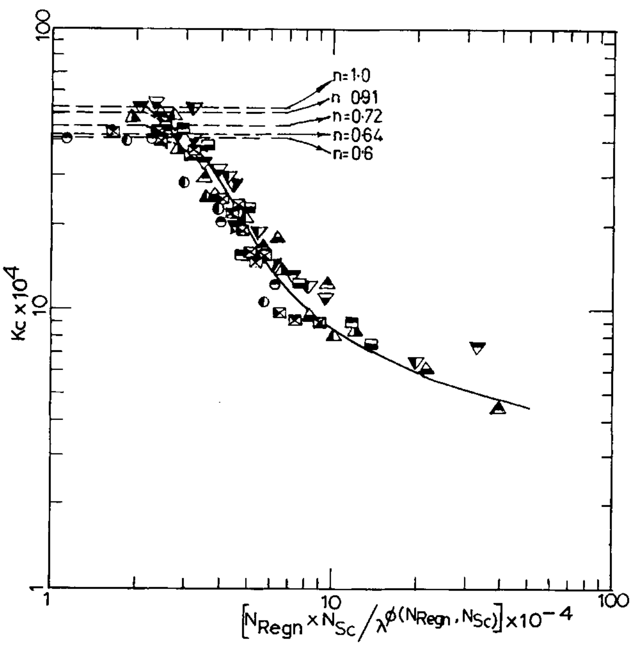

Fig. 6. K_C vs. $N_{Regn} \cdot N_{Sc}/\lambda^{\phi[N_{Regn}, N_{Sc}]}$. (—) present work, (- - -) Fan and Hwang[20] (for straight tubes).

Symbols

n/λ	10.5	30.0	101	220
0.60	◒		◐	
0.64	⊠	⊠	⊠	
0.72	⊟	◘		
0.91	▲	▲	▲	▲
1.0	▼	▼	▼	▼

Nomenclature

a	tube radius (cm)
C	cross-sectional mean concentration at the outlet (g/L)
D	effective diffusion coefficient (cm^2/s)
D_m	molecular diffusion coefficient (cm^2/s)
d_c	coil diam (cm)
d_t	tube diam (cm)
F	dimensionless concentration at the outlet
F_{exp}	experimental value of F, dimensionless
F_{theo}	theoretical value of F, dimensionless
h	coil pitch (cm)
K_c	dispersion coefficient ($D \cdot D_m/\bar{u}^2\, dt^2$), dimensionless
K_s	consistency index ($g \cdot s^{n-2}/cm$)
L	tube length (cm)
M	new dimensionless number reported by Mujawar and Raja Rao[5]
n	flow behavior index
n'	inverse of n
N_{Bo}	Bodenstein number ($=\bar{u}d_t/D_m$), dimensionless
N_{De}	Dean number ($=N_{Regn}/\sqrt{\lambda}$), dimensionless
N_{Re}	Reynolds number ($=d_t\,\bar{u}\rho/\mu$) dimensionless
N_{Regn}	generalized Reynolds number [$=(4n/3n+1)^n(1.0/8^{n-1})\,(d_t^n\bar{u}^{2-n}\rho/K_s)$], dimensionless

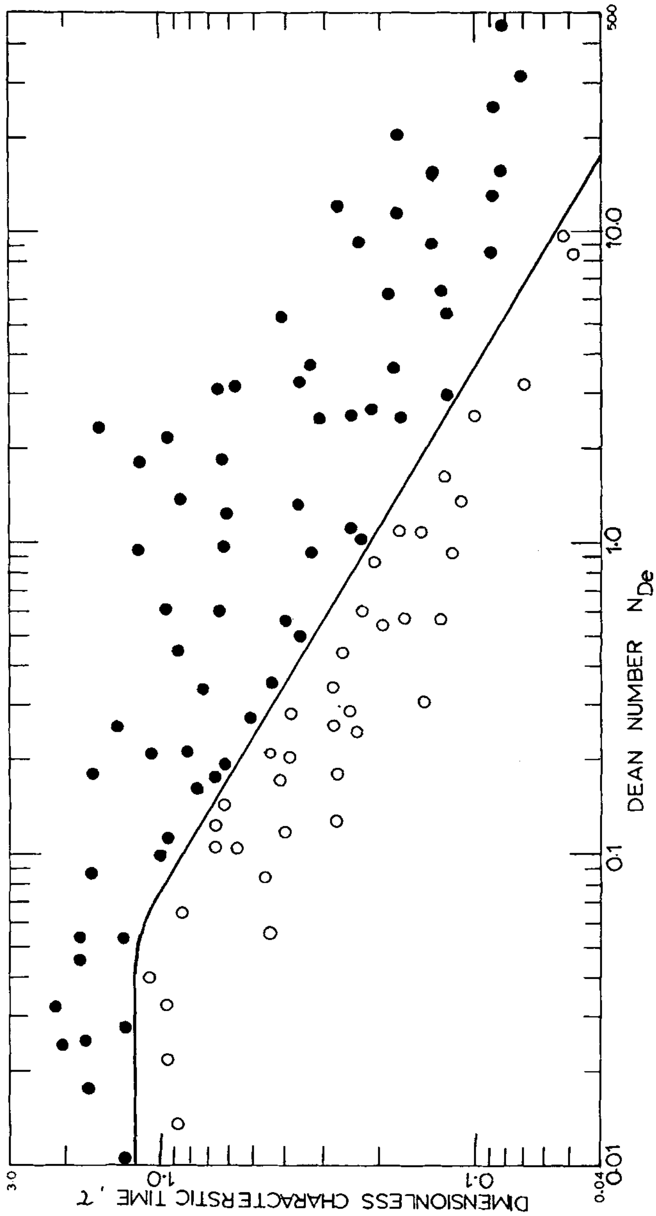

Fig. 7. τ vs. N_{De}. (●) Taylor's dispersion model is applicable; (○) Taylor's dispersion model is not applicable.

N_{Sc}	Schmidt number defined by eq. (7)
t	time (s)
$\bar{t}$	residence time ($=L/\bar{u}$) (s)
$\bar{u}$	average linear velocity of fluid (cm/s)
X	dimensionless distance in axial direction
λ	coil to tube diameter ratio ($=d_c/d_t$), dimensionless
μ	fluid viscosity, P
μ_a	apparent viscosity defined by eq. (8), P
ρ	fluid density (g/L)
θ	dimensionless time ($=t/\bar{t}$)
τ	dimensionless characteristic time ($=\bar{t}D_m/a^2$)
ξ	variable defined by eq. (10)
ψ	variable defined by eq. (11)

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